BE/Bi 101: Order-of-Magnitude Biology
Homework 7
Due date: Friday, February 27, 2015

"What interests me most is neither still life nor landscape: it is the human figure."

—Henri Matisse

1. Volumes of fluids.
In this problem, we will estimate where various fluids are in and out of your body.

a) Estimate the volume of air in one breath.

b) The atmosphere is about 21% oxygen by volume. The air you exhale is about 16% oxygen by volume. Estimate the total mass of oxygen that is taken into the body each day. Obviously, it cannot stay in the body, lest we blow up like a balloon or eventually become pure oxygen. Where does this oxygen go?

c) Estimate the total volume of blood in your body. (This will be useful for problem **??**.)

d) Estimate the amount of blood that is in your heart at any given time.

e) Estimate the volume of water you expell in urine each day.

f) Estimate the volume of saliva you produce each day.

2. Your growth.
In class, we derived a universal growth curve for animals (West, et al., *Nature*, **413**, 628–631, 2001). We found that

$$\frac{\mathrm{d}m}{\mathrm{d}t} = am^{\frac{3}{4}}\left(1-\left(\frac{m}{M}\right)^{\frac{1}{4}}\right), \tag{1}$$

where $m(t)$ is the mass of the growing organism and M is the mass of the fully grown adult. Since M is easily measured, the only parameter when trying to determine a curve is the constant a. We showed that this ODE has a solution, with $m(t=0)=m_0$,

$$r(t) \equiv \left(\frac{m}{M}\right)^{\frac{1}{4}} = 1-\left(1-\left(\frac{m}{M}\right)^{\frac{1}{4}}\right)\mathrm{e}^{-at/4M^{\frac{1}{4}}}. \tag{2}$$

a) Estimate a for your growth. Be sure to give the appropriate units.

b) The following table shows the value of a for various other mammals (West, et al., *Nature*, **413**, 628–631, 2001). Comment on how your estimated value compares to these. Make any other comments that come to mind about the significance of the values of a for all of these animals.

Organism	a ($\mathrm{g}^{\frac{1}{4}}$/day)
cow	0.28
pig	0.31
rabbit	0.36
guinea pig	0.21
rat	0.23
shrew	0.83

3. Number of capillaries.
Estimate the number of capillaries in your body. What is the total surface are of the capillaries? What is the total surface area of the aorta? Finally, estimate the ratio of the pressure differences (provided by the heart) needed to push blood through all the capillaries versus through the aorta. I.e., compute $\Delta p_{\mathrm{aorta}}/\Delta p_{capillaries}$.

Hints: Some of the scaling relationships we worked out in class about the vasculature may prove useful. The pictures in Fig. 1 should help you size up the aorta and capillaries. Finally, for estimating the pressure differences, remember that a pressure gradient drives flow against viscous dissipation. Find a scaling relation for the pressure gradient along the blood vessel, $\mathrm{d}p/\mathrm{d}x$, and then compute

$$\frac{\Delta p}{\ell} \approx \frac{\mathrm{d}p}{\mathrm{d}x}, \tag{3}$$

where ℓ is the length of the blood vessel.

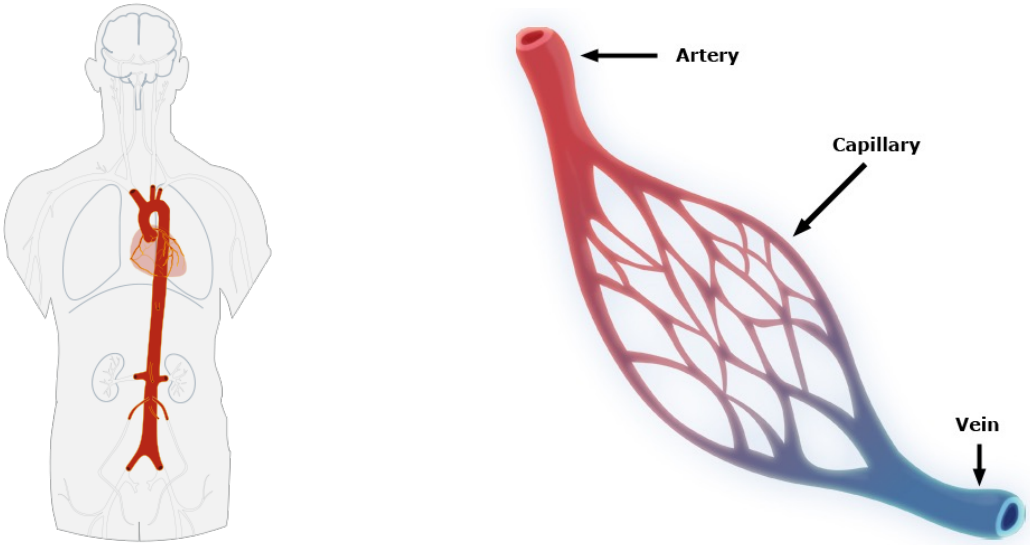

Figure 1: Schematics of the aorta and capillaries. Left, the aorta, in dark red. Right, a schematic of a small artery leading into several capillaries, and then into a vein. Both images are from Wikipedia.

4. Heisenberg's uncertainty.
The third episode of the TV show *Breaking Bad* opens with a flashback in which the main character, Walter White (later known as Heisenberg), and his research assistant, Gretchen, are writing the elemental components of the human body on a chalk board. They wrote down each element and then its atomic percentage in the human body. After writing down what they think is everything, Walter says they are "0.11958% shy." Heisenberg is uncertain about what is missing. Of course, it is silly that they somehow think they can be accurate enough for all of the elements they consider to add to *exactly* 100%.

We will try their problem with less dramatic effect, but with just as much fun. List what you think are the top ten most abundant elements in the human body *by mass*. Say why you think they are abundant, i.e., where they are found. Estimate the percent of the body mass taken by each of the top four (and then, by trivial extension, the rest of the elements combined). *Note*: This is *very* easy to look up. It is more fun to think about what is in your body first. You can look it up *after* your estimate and comment on any discrepencies you may see.

5. How quickly can Usain bolt? (20 pts. extra credit).
In class, we mentioned the fastest reaction time to a starting gun for a sprint. The IAAF, which governs the rules of track and field races, considers it a false start if a runner moves from the blocks within 100 milliseconds of the starting gun going off. Estimate the minimum time in which an athlete could move off the blocks after the gun goes off. How does this jibe with the 100 ms rule?