BE/Bi 101: Order-of-Magnitude Biology
Homework 6
Due date: Friday, May 20, 2016

"Life is the sum of trifling motions."

—Joseph Brodsky (1987 Nobel Prize in Literature, 1991 US Poet Laureate)

1. Noise in dim light.

The rod cells in the human eye convert the tiny energy imparted by being struck with a photon into a nervous impulse to the brain. You can imagine that to be able to do this, the rod cells are hypersensitive. In fact, the probability that a rod cell will respond to a single photon is about 0.1 (Phan, et al., *PRL*, **112**, 213601, 2014). You might also expect that a rod cell will occasionally send a signal to the brain in the absence of a photon. Say you are in a dim room where the probability that a photon arrived in a given second is low, say 0.1, and that we desire 99% certainty that a photon arrived when a signal was detected. How low must the probability that the rod cell produce a signal in a given second in the absence of a photon be in order to achieve this?

2. Thermal diffusion versus gravity.

In our discussions of random walks modeling diffusion, we neglected the influence of gravity. Imagine we have a particle of size a that is diffusing in a solvent. The difference in density between the particle and solvent is $\Delta\rho$; for simplicity in thinking about this problem, we will assume the particle is more dense than the solvent. You can imagine that if the particle is in a very tall tank of solvent, the particle will tend to settle, having higher probability of being toward the bottom of the tank. Conversely, if the tank of solvent is short, the particle can be anywhere within the tank with essentially equal probability. There is then some length scale, ℓ_{sed}, for which the thermal forces balance the gravitational forces.

Feric and Brangwynne (*Nat. Cell Biol.*, **15**, 1253–1259, 2013) found that nucleoli in *Xenopus* eggs are of higher density of the nucleoplasm, with $\Delta\rho \approx 0.03$ g/mL. The nucleus of *Xenopus* eggs is gigantic, about 450 µm across. How big would the nucleoli have to be to start to see sedimentation? Incidentally, nucleoli in the *Xenopus* egg can be over 2 µm in size.

3. Fidelity of translation.

In the process of translation, tRNAs bind to specific codons on the template mRNA. Because base pairing between complementary bases is more energetically favorable than base pairing between noncomplementary bases, we could reason that the correct tRNA is likely to bind to the codon and thereby incorporate the correct amino acid into the nascent protein. We will investigate this claim.

a) Estimate the pairing energy between a correct tRNA and a codon and between an incorrect tRNA and a codon.

b) Based on these energies, what is the probability of making an error? Does this jibe with the accuracy you would expect for translation? If not, please comment further on what this tells you about translation.

4. Efficiency in our eating.

In this problem, we will consider our energy consumption in terms of how much energy can be produced by plants.

a) Estimate how much carbon is incorporated into plants each year. This is called the net primary product, or npp.

b) Given that the biomass energy content is about 10 kJ per gram of carbon (how would you estimate this?), estimate the fraction of the npp that is consumed by humans if we directly consumed only plants.

c) Eating meat can be a terribly inefficient way of consuming the energy present in the npp. This is because the animal must be raised to maturity, and then killed. Estimate the amount of feed required to raise a chicken, hog, and cow for eating. *Hint*: This is a multistage estimate. You will need to think about the mass, lifetime, metabolic consumption, etc., of each animal.

d) Estimate the amount of energy a slaughtered chicken, pig, and cow respectively give to feeding humans.

e) Using the results you have gotten so far, compare the efficiency of consuming the respective animals compared to directly consuming plants.

5. Heisenberg's uncertainty.

The third episode of the TV show *Breaking Bad* opens with a flashback in which the main character, Walter White (later known as Heisenberg), and his research assistant, Gretchen, are writing the elemental components of the human body on a chalk board. They wrote down each element and then its atomic percentage in the human body. After writing down what they think is everything, Walter says they are "0.11958% shy." Heisenberg is uncertain about what is missing. Of course, it is silly that they somehow think they can be accurate enough for all of the elements they consider to add to *exactly* 100%.

We will try their problem with less dramatic effect, but with just as much fun. List what you think are the top ten most abundant elements in the human body *by mass*. Say why you think they are abundant, i.e., where they are found. Estimate the percent of the body mass taken by each of the top four (and then, by trivial extension, the rest of the elements combined). *Note*: This is *very* easy to look up. It is more fun to think about what is in your body first. You can look it up *after* your estimate and comment on any discrepancies you may see.